

## SUPERCONDUCTIVITY IN THE “HOT SPOTS” MODEL OF THE PSEUDOGAP STATE.

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**Abstract**—We analyze the anomalies of superconducting state in the model of pseudogap state induced by fluctuations of short – range order of “dielectric” (AFM (SDW) or CDW) type, and based on the scenario of “hot spots” formation on the Fermi surface, with the account of *all* Feynman graphs for electron interaction with pseudogap fluctuations, leading to strong scattering around the “hot spots”. We determine the dependence of superconducting critical temperature  $T_c$  on the effective width of the pseudogap, correlation length of short – range order and concentration of nonmagnetic impurities. We also discuss possible connection of these results with the general form of the phase diagram of superconducting cuprates.

One of the main problems in the physics of high – temperature superconducting cuprates remains the theoretical explanation of the typical phase diagram of these compounds [1]. Especially important is the clarification of the nature of the pseudogap state, observed in a wide region of temperatures and carrier concentrations [2], which is obviously crucial for the understanding of electronic properties both in normal and superconducting states. Despite the continuing discussions, the preferable “scenario” of the pseudogap formation seems to be based on the model of the strong scattering of electrons by antiferromagnetic (AFM, SDW) short – range order spin fluctuations [2,3].

In a recent paper [6] we have presented microscopic derivation of Ginzburg – Landau expansion and studied the influence of these pseudogap fluctuation on basic superconducting properties (for both *s*– and *d*-wave pairing) in the model of “hot spots” the Fermi surface. Similar analysis using Gorkov’s equations was given in Ref. [7].

In the model of nearly antiferromagnetic Fermi – liquid [4] electron interaction with spin fluctuations is usually described by dynamic susceptibility, characterized by correlation length  $\xi$  and frequency  $\omega_{sf}$  of spin fluctuations, which are to be determined from experiment and can depend both on carrier concentration and temperature. Both dynamic susceptibility and effective interaction are maximal (in momentum space representation) in the vicinity of vector  $\mathbf{Q} = (\pi/a, \pi/a)$  (*a* – lattice constant), which leads to the appearance of “two types” of quasiparticles — “hot” one with the momenta close to the points on the Fermi surface, connected by scattering vector  $\sim \mathbf{Q}$ , and “cold” one with the momenta close to the parts of this surface surrounding diagonals of the Brillouin zone [2,4,5].

For high enough temperatures, when  $2\pi T \gg \omega_{sf}$ , spin dynamics can be neglected [4] and electron interaction with spin (pseudogap) fluctuations reduces to scattering by appropriately defined static

Gaussian random field. For such model we can further simplify the form of effective interaction (correlator of the random field) [4,5], allowing the complete summation of Feynman perturbation series, leading to the following recurrence procedure determining single – electron Green’s function as  $G(\varepsilon_n \mathbf{p}) = G_{k=0}(\varepsilon_n \mathbf{p})$  from:

$$G_k(\varepsilon_n \mathbf{p}) = \frac{1}{i\varepsilon_n - \xi_k(\mathbf{p}) + ikv_k\kappa - \Sigma_k(\varepsilon_n \mathbf{p})} \quad (1)$$

$$\Sigma_k(\varepsilon_n \mathbf{p}) = W^2 s(k+1) G_{k+1}(\varepsilon_n \mathbf{p}) \quad (2)$$

where  $\kappa = \xi^{-1}$  — is the inverse correlation length of the pseudogap fluctuations,  $\varepsilon_n = 2\pi T(n+1/2)$ ,

$$\xi_k(\mathbf{p}) = \begin{cases} \xi_{\mathbf{p}+\mathbf{Q}} & \text{for odd } k \\ \xi_{\mathbf{p}} & \text{for even } k \end{cases} \quad (3)$$

$$v_k = \begin{cases} |v_x(\mathbf{p} + \mathbf{Q})| + |v_y(\mathbf{p} + \mathbf{Q})| & \text{for odd } k \\ |v_x(\mathbf{p})| + |v_y(\mathbf{p})| & \text{for even } k \end{cases} \quad (4)$$

where  $\mathbf{v}(\mathbf{p}) = \frac{\partial \xi_{\mathbf{p}}}{\partial \mathbf{p}}$  — is the velocity of quasiparticle with the spectrum  $\xi_{\mathbf{p}}$ , which is taken in the standard form [4]:

$$\xi_{\mathbf{p}} = -2t(\cos p_x a + \cos p_y a) - 4t' \cos p_x a \cos p_y a - \mu \quad (5)$$

where  $t$  – is transfer integral between nearest, while  $t'$  – between second nearest neighbors on the square lattice,  $\mu$  – is the chemical potential. Parameter  $W$  with dimension of energy determines the effective width of the pseudogap. It is clear that both  $W$  and correlation length  $\xi$  are (within our semiphenomenological approach) some functions of carrier concentration (and probably temperature) to be determined from the experiment [4,5].

The value of  $s(k)$  is determined by combinatorics of Feynman diagrams and for the case of Heisenberg spin (SDW) fluctuations [4] is equal to:

$$s(k) = \begin{cases} \frac{k+2}{3} & \text{for odd } k \\ \frac{k}{3} & \text{for even } k \end{cases} \quad (6)$$

The limits of applicability of our approximations were discussed in detail in Refs. [4,5].

Remarkable advantage of our model is the possibility of complete summation of *all* Feynman diagrams (including those with intersecting interaction lines) also for the vertex parts, determining the response functions to an arbitrary external perturbation. Detailed enough discussion of the vertex parts was presented in Ref. [8]. Here we just write down the appropriate recurrence relations appearing after the appropriate analysis for “triangular” vertices in Cooper channel, analogous to those derived in Ref. [6] and describing the response to an arbitrary fluctuation of superconducting order parameter (energy gap)  $\Delta_{\mathbf{q}}e(\mathbf{p})$ , where the symmetry factor determining the type (symmetry) of pairing is taken here for the case of *d*-wave pairing as  $e(\mathbf{p}) = \cos p_x a - \cos p_y a$ , and we assume the usual case of singlet pairing. The vertex of interest to us can be written as:

$$\Gamma(\varepsilon_n, -\varepsilon_n, \mathbf{p}, -\mathbf{p} + \mathbf{q}) \equiv \Gamma_{\mathbf{p}}(\varepsilon_n, -\varepsilon_n, \mathbf{q})e(\mathbf{p}) \quad (7)$$

Then  $\Gamma_{\mathbf{p}}(\varepsilon_n, -\varepsilon_n, \mathbf{q})$  is determined by the recurrence procedure of the following form:

$$\begin{aligned} \Gamma_{\mathbf{p}k-1}(\varepsilon_n, -\varepsilon_n, \mathbf{q}) = 1 - W^2 r(k) G_k(\varepsilon_n, \mathbf{p} + \mathbf{q}) \times \\ \times G_k(-\varepsilon_n, \mathbf{p}) \Xi_k(\varepsilon_n, \mathbf{p}, \mathbf{q}) \Gamma_{\mathbf{p}k}(\varepsilon_n, -\varepsilon_n, \mathbf{q}) \end{aligned} \quad (8)$$

where

$$\begin{aligned} \Xi_k(\varepsilon_n, \mathbf{p}, \mathbf{q}) = \left\{ 1 + \right. \\ \left. + \frac{2ik\kappa v_k}{G_k^{-1}(\varepsilon_n, \mathbf{p} + \mathbf{q}) - G_k^{-1}(-\varepsilon_n, \mathbf{p}) - 2ik\kappa v_k} \right\} \quad (9) \end{aligned}$$

“Physical” vertex is given by  $\Gamma_{\mathbf{p}k=0}(\varepsilon_n, -\varepsilon_n, \mathbf{q})$ . Additional combinatorial factor  $r(k)$  for the case of Heisenberg spin (SDW) fluctuations [4] is given by:

$$r(k) = \begin{cases} k & \text{for even } k \\ \frac{k+2}{9} & \text{for odd } k \end{cases} \quad (10)$$

The choice of the sign before the factor of  $W^2$  in the right hand side of Eq. (8) depends on the symmetry of superconducting order parameter and the nature of pseudogap fluctuations [6], here our choice again corresponds to the case of Heisenberg spin (SDW) fluctuations [10].

Scattering by normal (non magnetic) impurities can easily be accounted for by self – consistent Born approximation writing down “Dyson equation” for

single – electron Green’s function generalizing Eq. (1) and shown diagrammatically in Fig. 1 (a), where we have introduced:

$$G_{0k}(\varepsilon_n \mathbf{p}) = \frac{1}{i\varepsilon_n - \xi_k(\mathbf{p}) + ikv_k\kappa} \quad (11)$$

so that instead of (2) we get:

$$\Sigma_k(\varepsilon_n \mathbf{p}) = \rho U^2 \sum_{\mathbf{p}} G(\varepsilon_n \mathbf{p}) + W^2 s(k+1) G_{k+1}(\varepsilon_n \mathbf{p}) \quad (12)$$

where  $\rho$  – is concentration of point – like impurities with potential  $U$ , and impurity contribution to self – energy contains the fully dressed Green’s function  $G(\varepsilon_n \mathbf{p})$ , to be determined self – consistently.

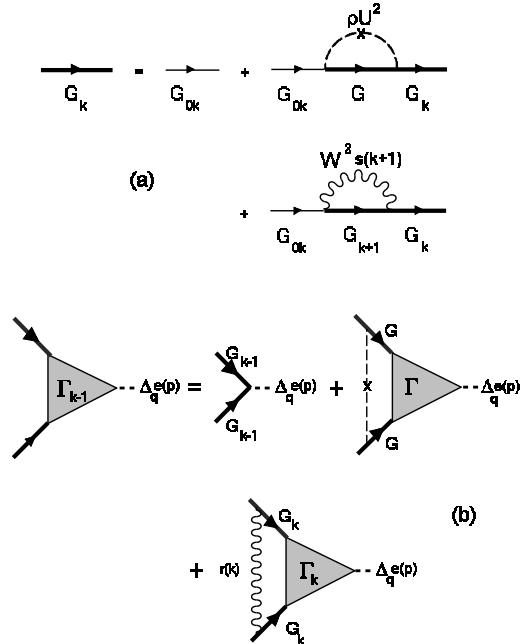


FIG. 1. Recurrence relations for the Green’s function (a) and “triangular” vertex (b) with the account of scattering by random impurities.

In comparison with impurity free case, we have an obvious substitution (renormalization):

$$\varepsilon_n \rightarrow \varepsilon_n - \rho U^2 \sum_{\mathbf{p}} \text{Im} G(\varepsilon_n \mathbf{p}) \quad (13)$$

Dropping full self – consistency of self – energy over impurity scattering we obtain just the usual substitution:

$$\varepsilon_n \rightarrow \varepsilon_n + \gamma_0 \text{sign} \varepsilon_n \quad (14)$$

where  $\gamma_0 = \pi \rho U^2 N_0(0)$  – is standard Born impurity scattering rate ( $N_0(0)$  – free electron density of states at the Fermi level).

For a “triangular” vertex with the account of impurity scattering we obtain a recurrence relation of general form shown graphically in Fig.1 (b). However for the case of the vertex, describing interaction with a fluctuation of superconducting order parameter with  $d$ -wave symmetry this equation is simplified considerably as the second term in the right hand side of Fig. 1 (b) is in fact zero due to  $\sum_{\mathbf{p}} e(\mathbf{p}) = 0$  (cf. similar situation discussed in Ref. [9]). Then our recursion relation for the vertex part reduces to (8), where  $G_k(\pm \varepsilon_n \mathbf{p})$  are determined by Eqs. (11), (12), i.e. are just Green’s function “dressed” by impurity scattering and defined diagrammatically in Fig. 1 (a).

Superconducting transition temperature is determined by the usual equation for Cooper instability of the normal phase:

$$1 - V\chi(0; T) = 0 \quad (15)$$

where the generalized Cooper channel susceptibility is given by:

$$\begin{aligned} \chi(\mathbf{q}; T) = & -T \sum_n \sum_{\mathbf{p}} e^2(\mathbf{p}) \times \\ & \times G(\varepsilon_n \mathbf{p} + \mathbf{q}) G(-\varepsilon_n, -\mathbf{p}) \Gamma_{\mathbf{p}}(\varepsilon_n, -\varepsilon_n, \mathbf{q}) \end{aligned} \quad (16)$$

Pairing interaction constant  $V$  is, as usual in BCS – like approach, assumed to be non zero in some layer of the width of  $2\omega_c$  around the Fermi level and determines the “bare” transition temperature  $T_{c0}$  in the absence of pseudogap fluctuations by the standard BCS equation [6]. Then, choosing rather arbitrarily e.g.  $\omega_c = 0.4t$  and  $T_{c0} = 0.01t$  we can easily find the appropriate value of  $V$  corresponding to this value of  $T_{c0}$ . Actually this can be easily done for any other choice of parameters of our model.

Knowledge of Green’s functions and vertices allows us to derive Ginzburg – Landau expansion in a standard way [6]. Then, using microscopic values of Ginzburg – Landau coefficients we can determine all characteristics of a superconductor close to  $T_c$  (e.g. coherence length, penetration depth, specific heat discontinuity etc.). Appropriate results will be given in a more detailed publication [10].

Numerical calculations were performed for different values of  $t$ ,  $t'$ , and  $\mu$  of the “bare” electronic spectrum and also for different values of  $W$  and inverse correlation length  $\kappa a$ .

In Fig. 2 we show the dependence of superconducting critical temperature  $T_c$  on the effective

width of the pseudogap  $W$  for different values of impurity scattering rate. It is seen that pseudogap fluctuations lead to significant suppression of superconductivity and that in case of finite impurity scattering we always obtain some “critical” value of  $W$ , corresponding to complete disappearance of  $T_c$ . This  $T_c$  suppression is obviously due to partial “dielectrization” of electronic spectrum around the “hot spots” on the Fermi surface [4,5].  $T_c$  dependence on the value of correlation length of short – range order (pseudogap) fluctuations is much slower, in all cases the growth of  $\xi$  (diminishing  $\kappa$ ) enhances the effect of pseudogap fluctuations (suppression of  $T_c$ ). We drop the appropriate results to spare space.

Now we can move to an attempt to model a typical phase diagram of a cuprate superconductor within “hot spots” model of the pseudogap state. First attempt of such modelling in an oversimplified (“toy”) version of our model was undertaken in Ref. [11]. The main idea is to identify our parameter  $W$  with experimentally observable effective width of the pseudogap (temperature of crossover to the pseudogap region of the phase diagram)  $E_g \approx T^*$ , which is determined from numerous experiments [1–3]. It is well known that the value of this parameter actually drops, more or less linearly with concentration of doping impurity (carrier concentration), from values of the order of  $10^3$ K, becoming zero at some critical concentration  $x_c \approx 0.19..0.22$ , slightly greater than the “optimal” concentration  $x_o \approx 0.15..0.17$  [1,12]. Accordingly we just assume similar concentration dependence of our effective pseudogap width  $W(x)$ . In this sense we can consider our  $W(x)$  as determined from the experiment. Then the only “fitting” parameter of the model is concentration dependence of the “bare” superconducting transition temperature  $T_{c0}(x)$ , which would have been existent in the absence of pseudogap fluctuations. Unfortunately, as already has been noted in Ref. [6], this dependence of  $T_{c0}(x)$  is, in general case, unknown and is not determined from any known experiment.

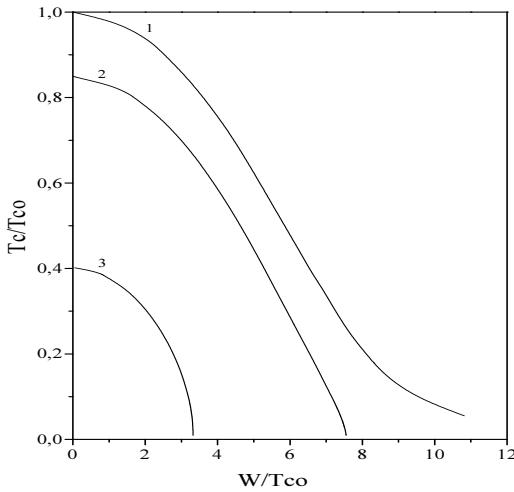


FIG. 2.  $T_c$  dependence on the effective width of the pseudogap  $W$  for the case of  $d$ -wave pairing and different values of impurity scattering rate  $\gamma_0/T_{c0}$ : 0 – 1; 0.18 – 2; 0.64 – 3. Inverse correlation length  $\kappa a=0.2$ .

In the framework of our BCS – like approach any significant concentration dependence of the “bare”  $T_{c0}$  seems to be unrealistic<sup>1</sup>. So we just assume there is no dependence of  $T_{c0}$  on  $x$  at all, but take into account the fact, that introduction of any doping impurity obviously leads to appearance of some impurity scattering (internal disorder), which can be described by the appropriate linear dependence of  $\gamma(x)$ . Assume that it is this growth of disorder is what leads to the complete suppression of  $d$ -wave pairing (according to the well known Abrikosov – Gorkov dependence, cf. e.g. Ref. [9]) say at  $x = 0.3$ . Then our approach allows to calculate concentration dependence  $T_c(x)$  for all values of  $x$ . Results of such calculation with parameters more or less appropriate for  $La_{2-x}Sr_xCuO_4$ , in case of Heisenberg (SDW) pseudogap fluctuations, and with the account of this very simple model of impurity scattering, are shown in Fig. 3. Actual values of parameters of the model used in these calculations are shown at the same figure. “Experimental” data for  $T_c(x)$  shown in this figure with “diamonds” are obtained from an empirical relation [12,13]:

$$\frac{T_c(x)}{T_c(x=x_o)} = 1 - 82.6(x - x_o)^2 \quad (17)$$

which gives rather good fit to experimentally ob-

served concentration dependence of  $T_c$  for a number of cuprates. We can see that in the whole underdoped region our model gives practically ideal fit to “experimental” data, assuming quite reasonable values of  $W(x)$ <sup>2</sup>.

It is interesting to analyze dependence of superconducting transition temperature  $T_c$  under additional disordering of the system at different compositions (concentration of carriers). Such disordering was studied in a number of experiments, e.g. introducing disorder by neutron irradiation [14] or by chemical substitutions [15], in our model it can be simulated by additional impurity scattering, characterized by an extra parameter  $\gamma_0$ , which is just added to our parameter of an internal disorder  $\gamma(x)$ . Results of this type of calculations for two values of this additional scattering parameter are also shown in Fig. 3. It is seen that in complete accordance with experiments of Ref. [15], introduction of additional “impurities” (disorder) leads to rather fast reduction of superconducting region on the phase diagram. Also we note that in accordance with experiments [14,15], suppression of superconductivity by disorder in the underdoped region of the phase diagram (pseudogap state) is significantly faster, than at optimal dopings.

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<sup>2</sup>Note that in case of Ising like pseudogap fluctuations we need rather unrealistic values of  $W(x = 0)$ , order a magnitude larger, than in Heisenberg case. However, CDW like fluctuations can also give rather good fit [10].

<sup>1</sup>This can be only due to some relatively smooth concentration dependence of the density of states at the Fermi level.

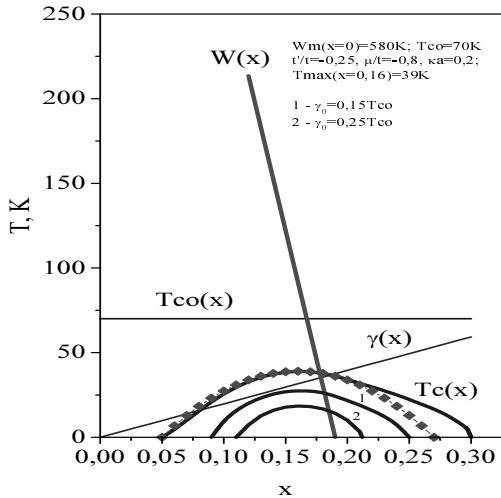


FIG. 3. Model phase diagram for  $La_{2-x}Sr_xCuO_4$  - type system in the case of concentration independent “bare”  $T_{co}$  with “internal” disorder  $\gamma(x)$  linear in  $x$ .

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